St Andrew's Church of England Primary School

Progression in Calculation

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

National Curriculum 2014

Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Introduction

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly.

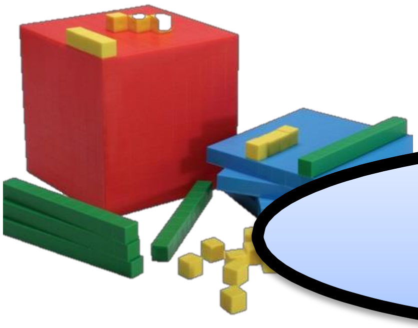
Strategies for calculation need to be represented by models and images to support, develop and secure understanding.

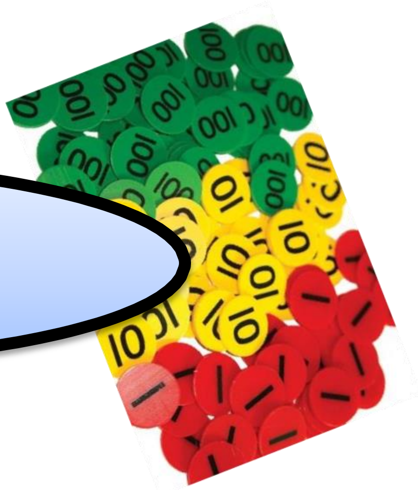
When teaching a new strategy it is important to start with numbers that the pupil can easily manipulate so that they can understand the methodology.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

The quality and variety of mathematical vocabulary pupils hear and speak are key factors in developing their mathematical justification, argument and proof. They must be assisted in making their thinking clear to themselves as well as others. Teachers should ensure that pupils build secure foundations by using discussion to probe and remedy their misconceptions.

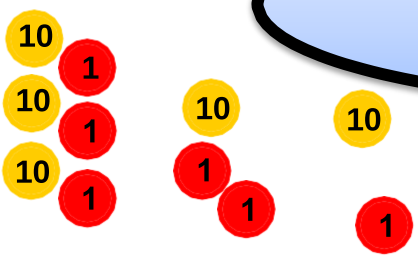
Structuring Learning



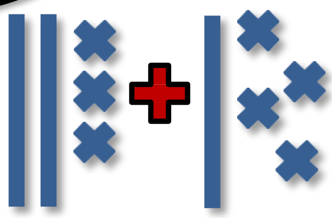


Concrete

Pictorial



33 − 12 = 21



Abstract

15 + 7 = 22

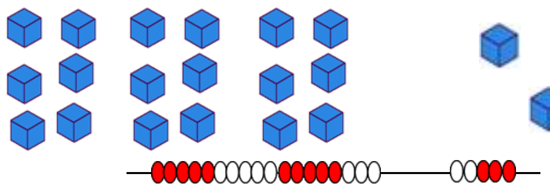
24 – 8 = 16

Facilitating Learning

Concrete

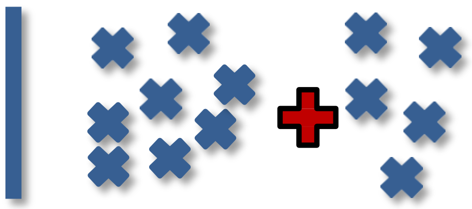
Pupils should have access to a range of suitable manipulatives so that they can explore concepts in ways that make sense to them.

e.g. **18+5**



Pictorial

Pupils should explore how they can record their concrete representations in a way that they understand.



Abstract

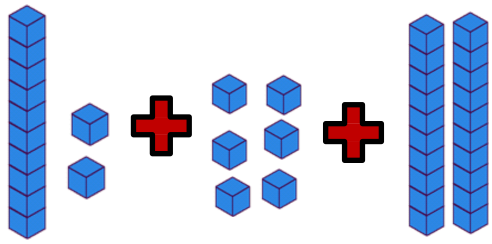
Pupils use their own written or mental strategies that do not rely upon a visual representation.

It is important for children to see calculations written in different ways. They must understand that '=' is 'equal to' and can appear in different places in a number sentence.

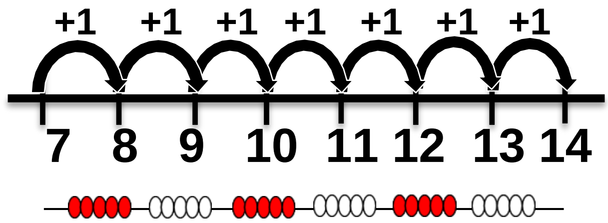
18 + 5 = 23	**18 + 5 = 10 + 13**
7 + 9 = [ ] **7 = [ ] - 9**	**7 + 2 = [ ] - 7**

Structures of Addition

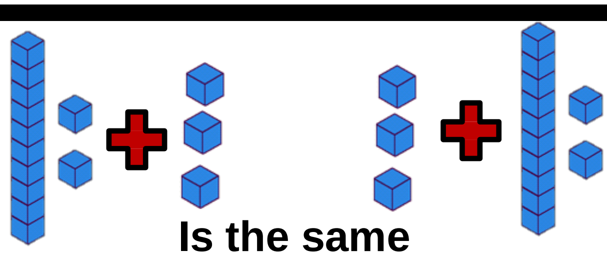
Aggregation
Combine two or more sets
Total
“How many/much altogether?”



Augmentation
Increasing
Go up by
“Start at___and count on”



Commutative
Understanding addition can be done in any order
“Start with the bigger number when counting on”



Is the same as/equal to (=)

Addition

Stage 1

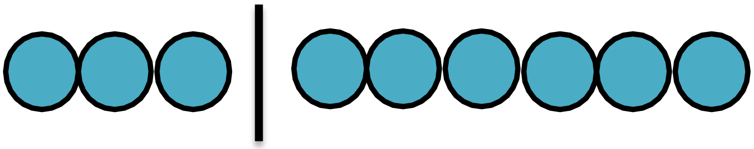
Pupils must experience combining two, then more, groups of objects using counting on.

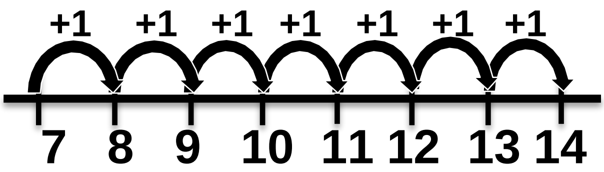
Concrete

- Using concrete apparatus to first **count all** and then by **counting on** from the largest number.

Pictorial

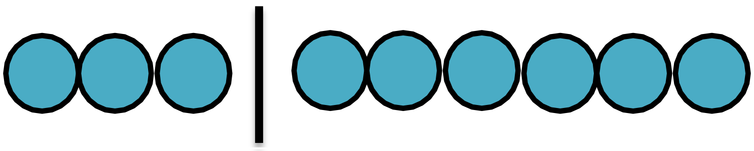
- Record building visual images e.g.





Abstract

- Progressing to recording number sentences alongside e.g.



3 and 6

3 + 6

Stage 2

Pupils should be able to partition numbers in different ways e.g.

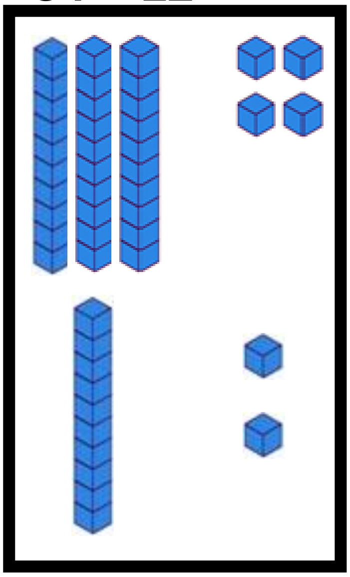
5 as 2 + 2 + 1 or 4 + 1

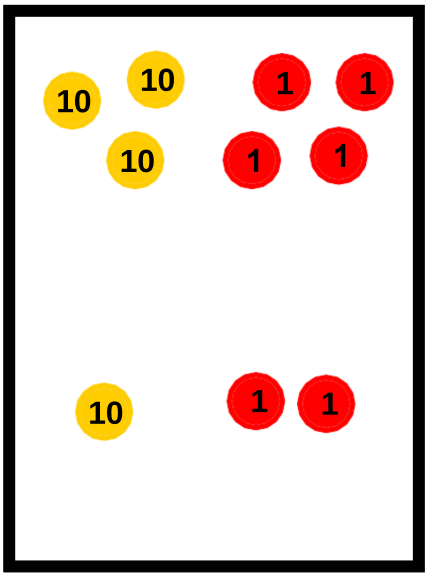
27 as 20 + 7 or 10 + 17

Concrete

Using equipment to actively demonstrate partitioning e.g.

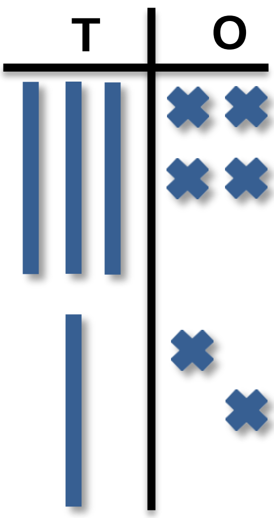
34 + 12 =





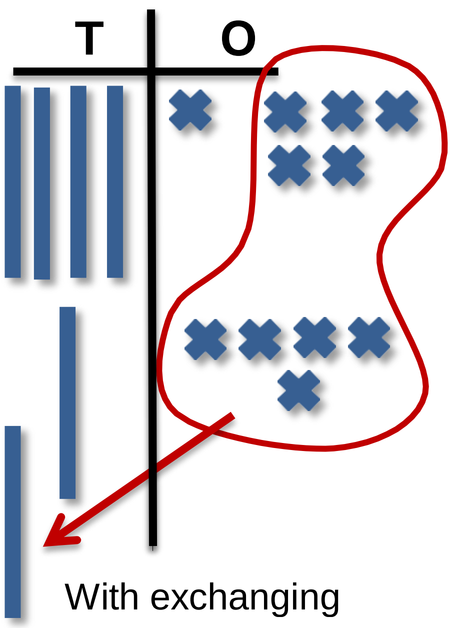
Pictorial

34 + 12 =



Without exchanging

46 + 15 =



With exchanging

Abstract

44 + 15 = 59

40 4 10 5

40 + 10 = 50

4 + 5 = 9

50 + 9 = 59

Without exchanging

27 + 16 = 43

20	7
30	13
10	6

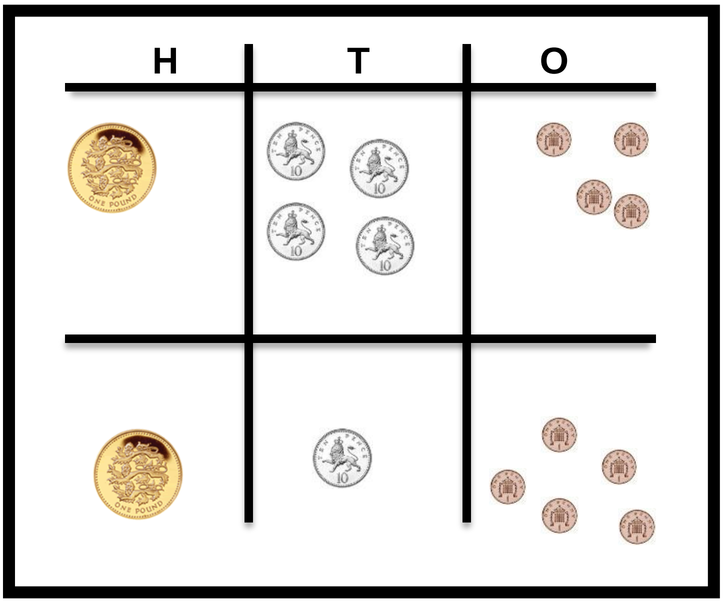
30 + 13

With exchanging

Stage 3

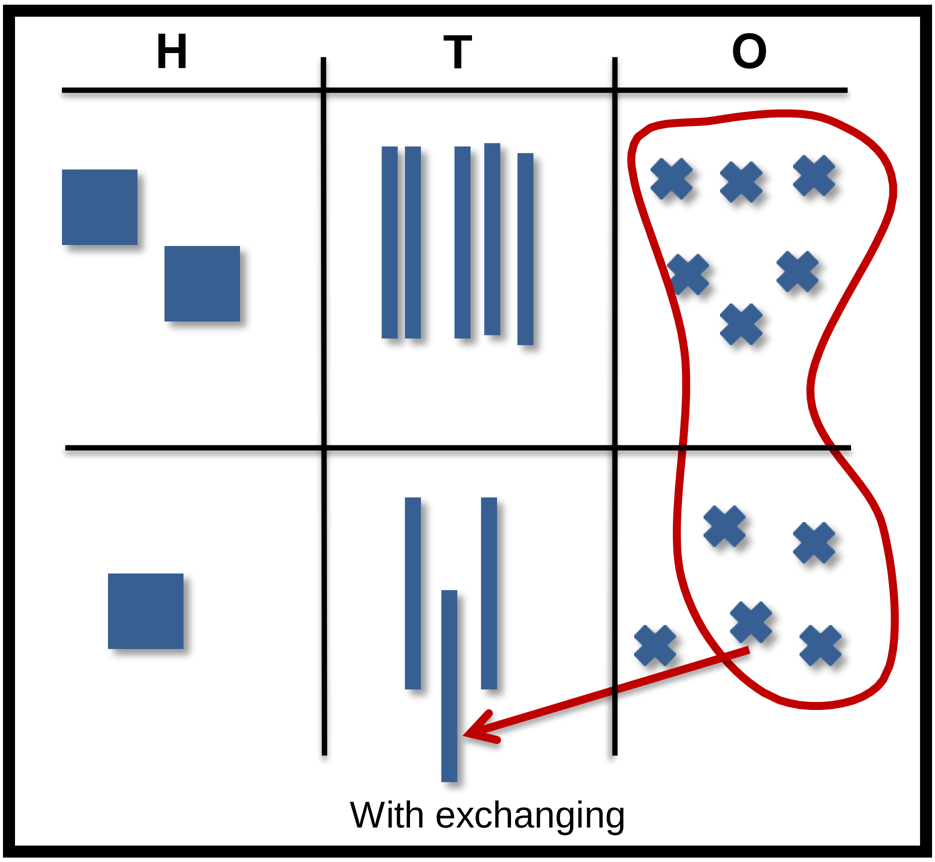
Concrete

144 + 115 =



Pictorial

256 + 125 =



Abstract

Expanded column method

$$
\begin{array}{ccccc}
\textbf{H} & & \textbf{T} & & \textbf{O} \\
100 & + & 40 & + & 4 \\
100 & + & 10 & + & 5 \\
\hline
200 & + & 50 & + & 9 \\
 & & & & = 259
\end{array}
$$

Without exchanging

$$
\begin{array}{ccc}
\textbf{T} & & \textbf{O} \\
40 & + & 6 \\
10 & + & 5 \\
\hline
\cancel{5}0 & + & 11
\end{array}
$$

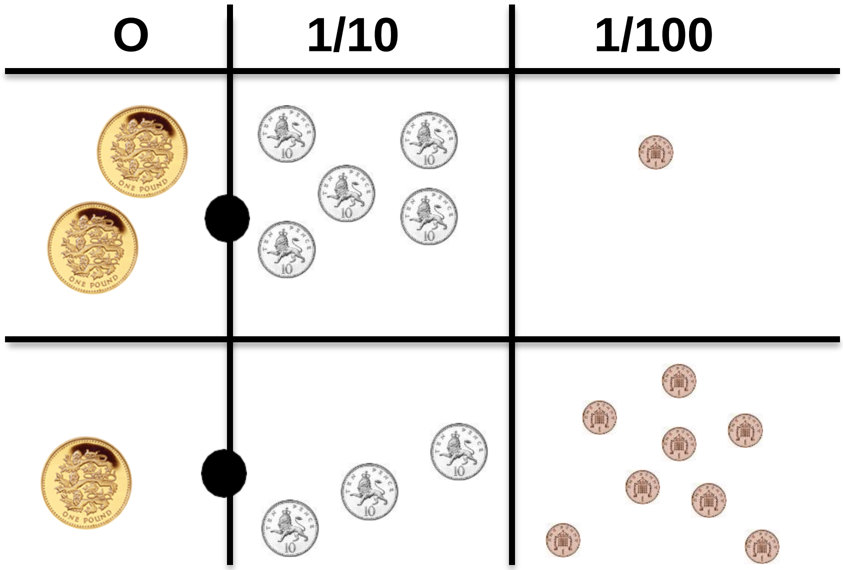
$$60 \quad + \quad 1 \quad = 61$$

With exchanging

Stage 4

Concrete

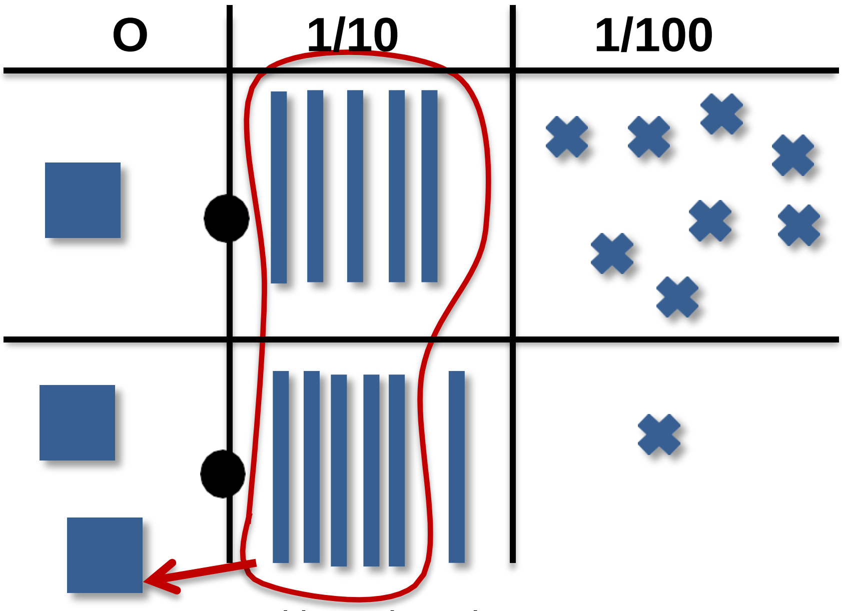
2.51 + 1.38 =



Without exchanging

Pictorial

1.58 + 1.61 =



With exchanging

Abstract

Compact column method

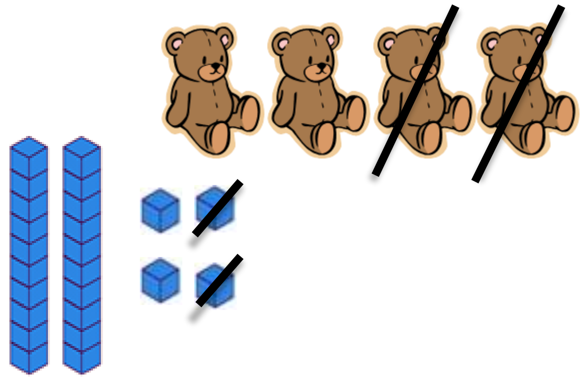
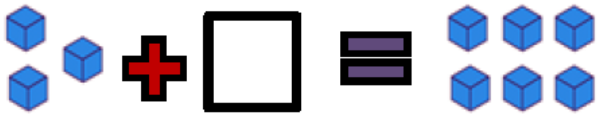
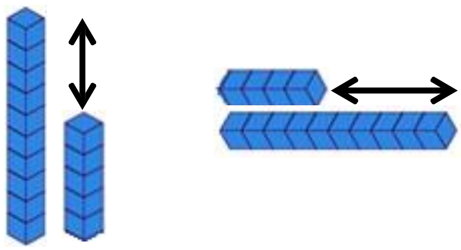
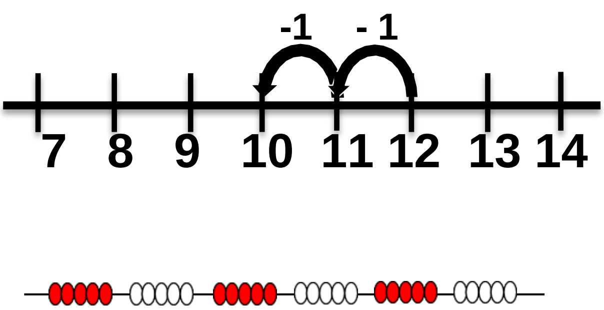
$$\begin{array}{r} 144 \\ +\ \ \ \ \\ 115 \\ \hline 259 \end{array}$$

Without exchanging

$$\begin{array}{r} 569 \\ +\ \ \ \ \\ 752 \\ \hline 1321 \\ {\scriptstyle 1\ 1\ \ } \end{array}$$

With exchanging

Structures of Subtraction

Partitioning Take away	
Inverse Addition How many more are needed?	
Comparison What is the difference?	
Reduction Count back	

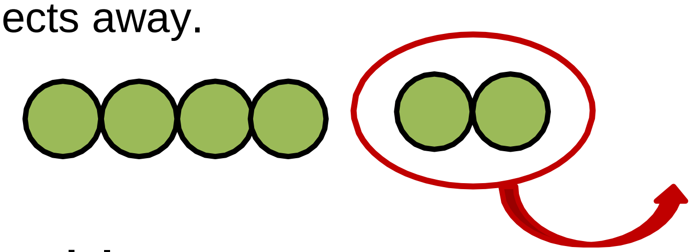
Subtraction

Stage 1

Pupils must experience finding out how many are left.

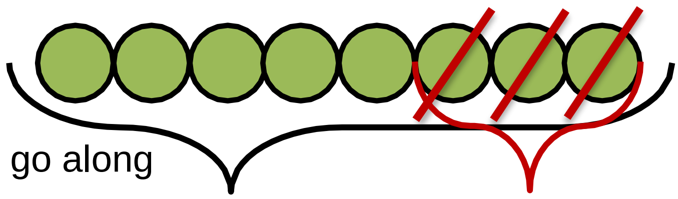
Concrete

Pupils should begin to use apparatus to actively take a number of objects away.

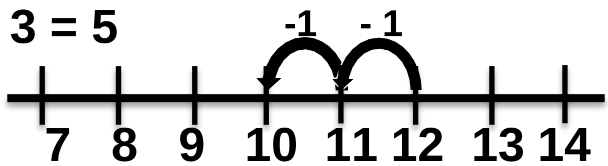


Pictorial

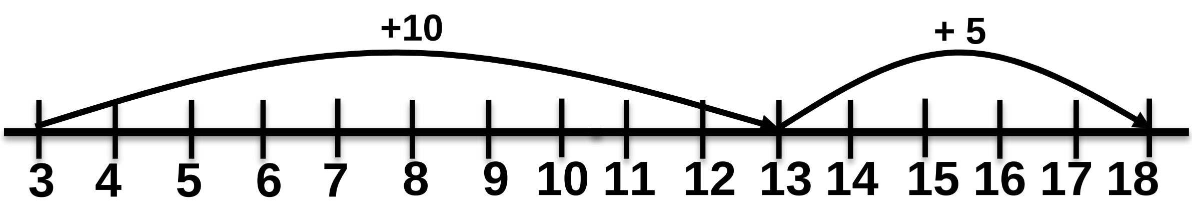
Pupils should being to record building visual images e.g.



8 - 3 = 5



Counting back



Counting on

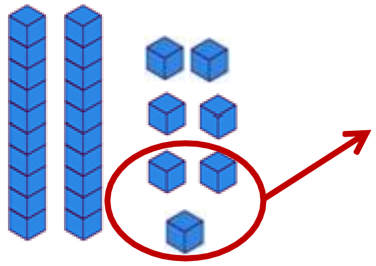
Abstract

18 – 3 = 15

Stage 2

Pupils should begin to exchange numbers.

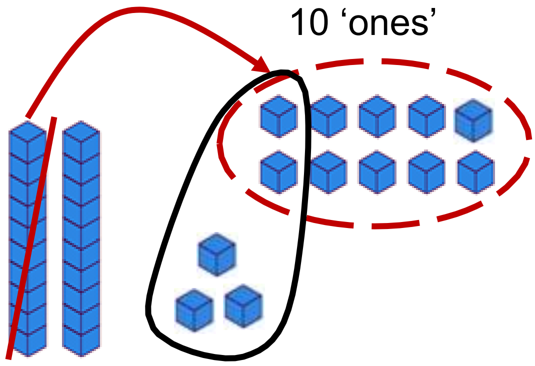
Concrete



27 – 3 =

Without exchanging

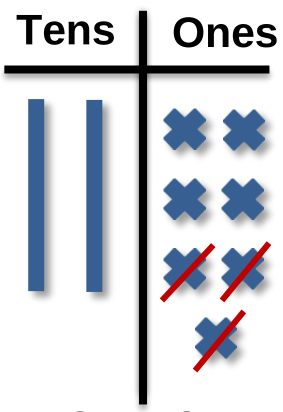
Exchange ‘ten’ for
10 ‘ones’



23 – 5 =

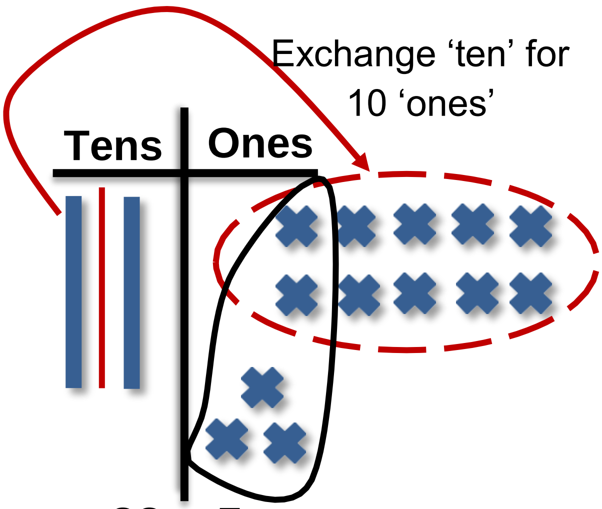
With exchanging

Pictorial



27 – 3 =

Without exchanging



23 – 5 =

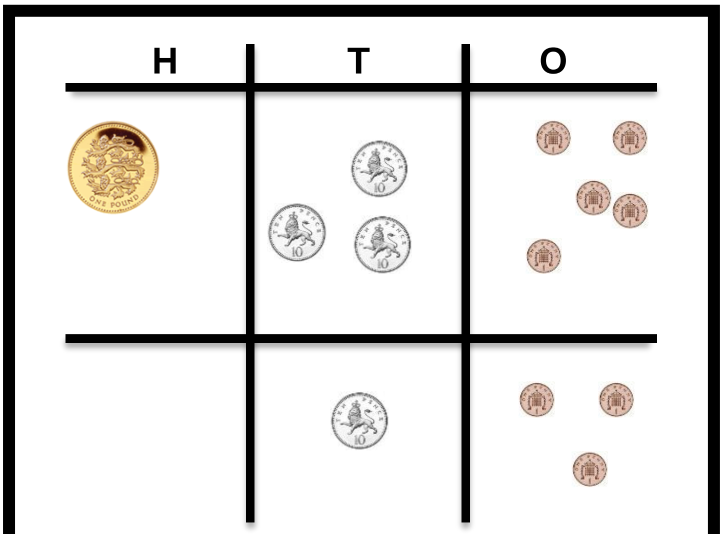
With exchanging

Stage 3

Pupils record calculations as expanded column subtraction using partitioning.

Concrete

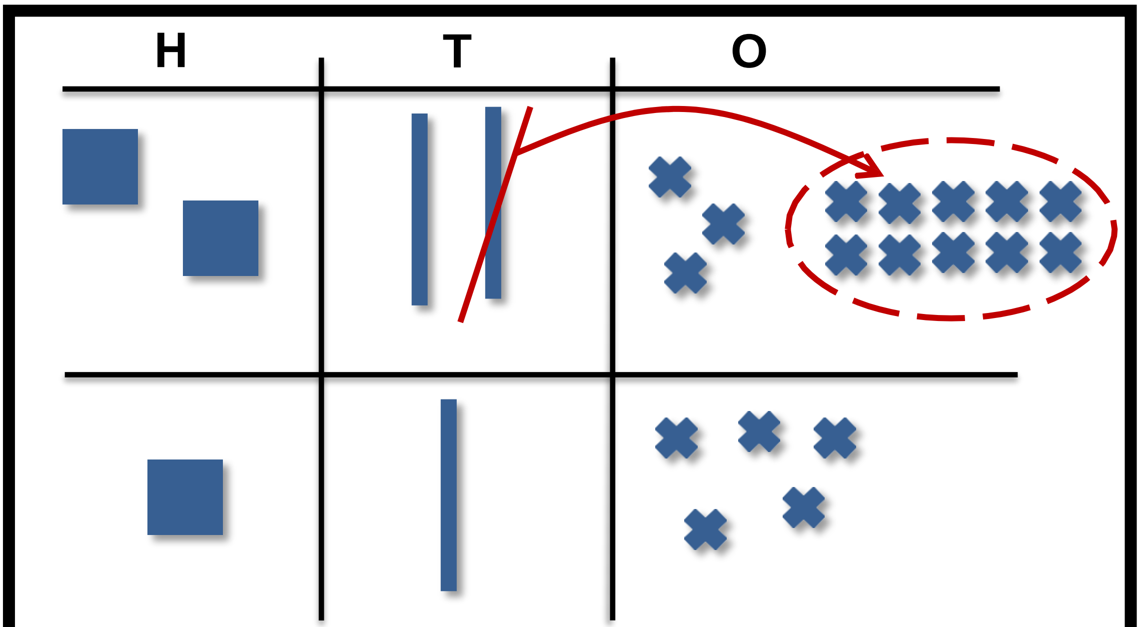
135 – 13 =



Without exchanging

Pictorial

223 – 115 =



With exchanging

Abstract

	T	O	
	70	4	
-	20	1	
	50	3	= 53

	H	T	O	
	100	40	7	
-	100	30	2	
	0	10	5	= 15

Without exchanging

	T	O	
	40		
	~~50~~	$^{1}2$	
-	10	7	
	30	5	= 35

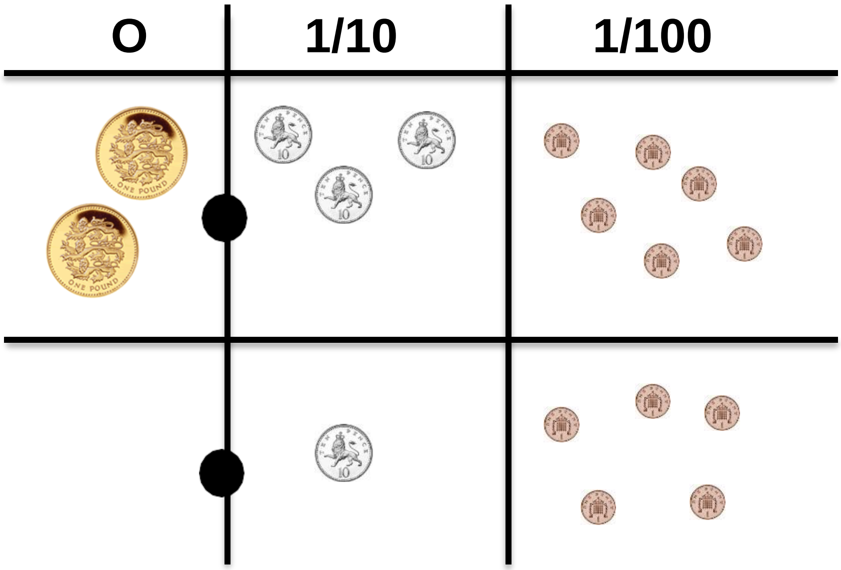
	H	T	O	
	400	1 10		
	~~500~~	~~20~~	$^{1}1$	
-	100	50	8	
	300	60	3	= 363

With exchanging

Stage 4

Concrete

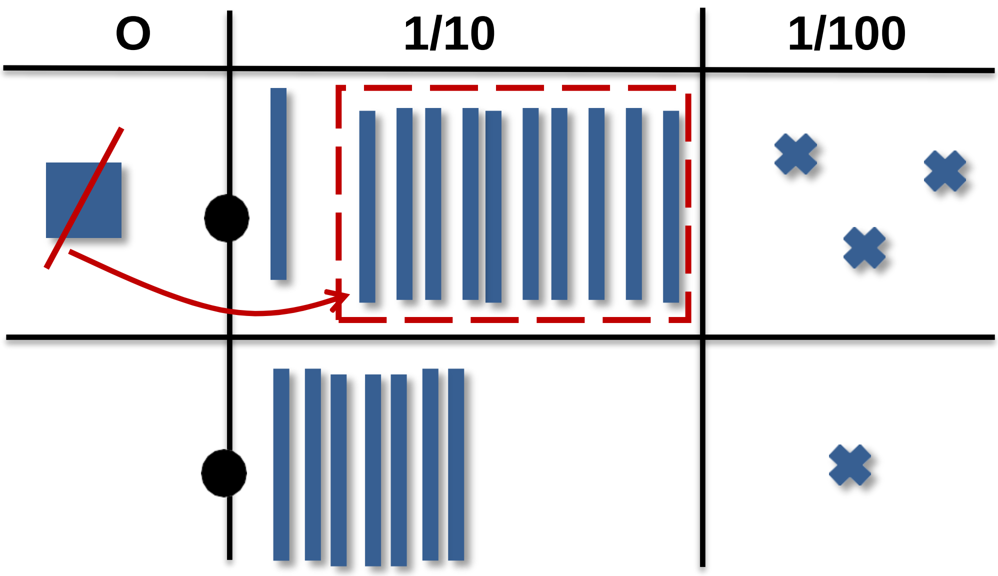
2.36 – 0.15 =



Without exchanging

Pictorial

1.13 – 0.71 =



With exchanging

Abstract

$$\begin{array}{r} 1358 \\ -\quad 146 \\ \hline 1212 \end{array}$$

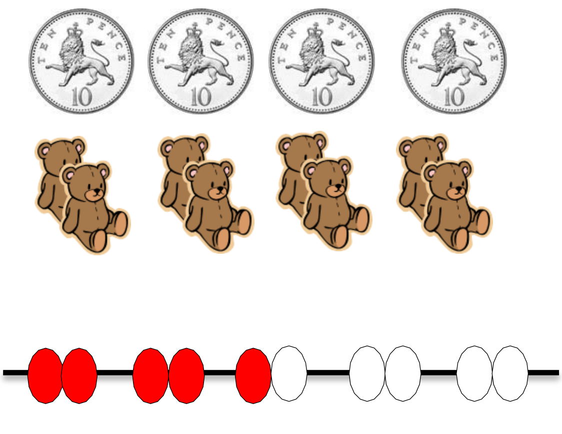
Without exchanging

$$\begin{array}{r} {}^{6}\cancel{7}\,{}^{\overset{1}{0}}\cancel{1}\,{}^{1}2\,6 \\ -\quad 3134 \\ \hline 3992 \end{array}$$

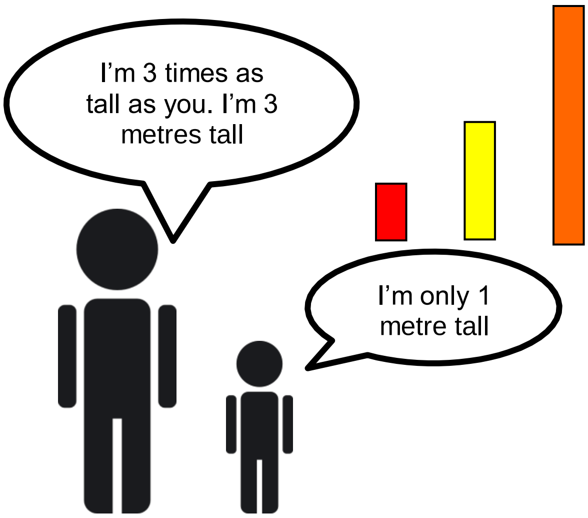
With exchanging

Structures of Multiplication

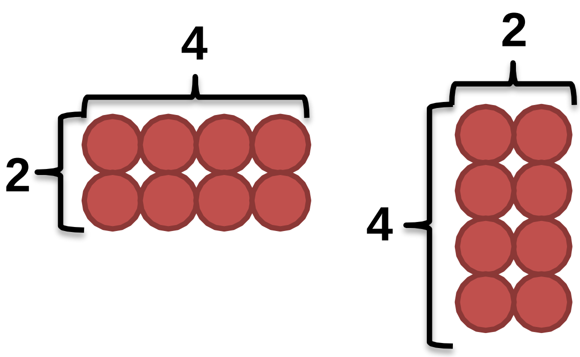
Repeated Addition
How many lots (sets)



Scaling
Scale factor
Doubling, trebling
How many times



Commutative Law
Scale factor
Doubling, trebling
How many times



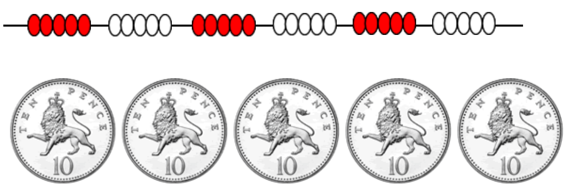
2 x 4 is the same as/equal to 4 x 2

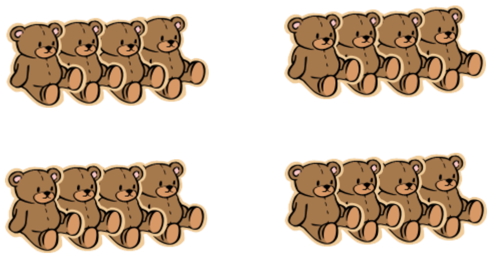
Stage 1

Pupils should recognise how grouping similar objects can support calculations.

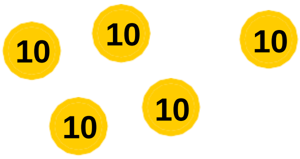
Concrete

Pupils being using manipulatives to represent groups or ‘lots of’ objects.

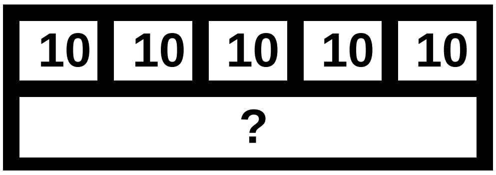




Pictorial



5 x 10 =



5 x 10 =

Abstract

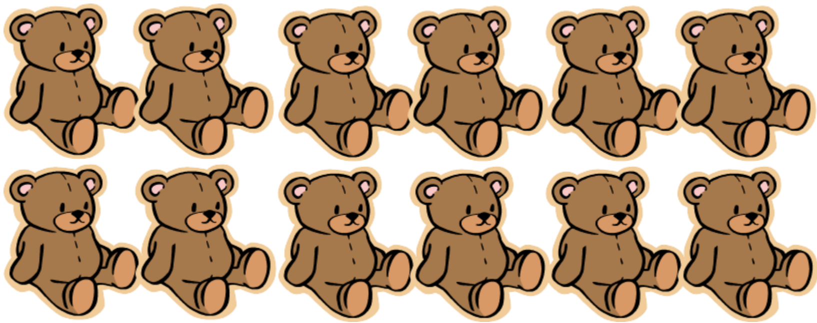
2 + 2 + 2 + 2 = 8

2 x 4 = 8

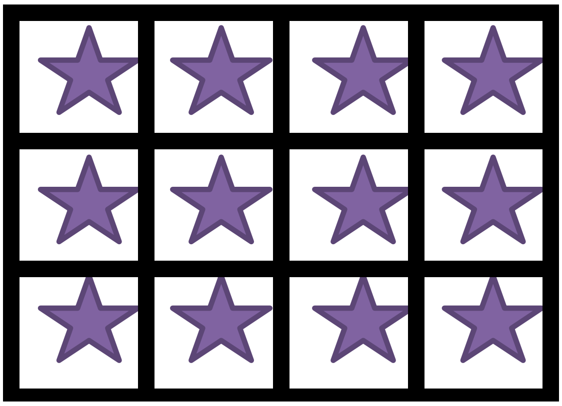
Stage 2

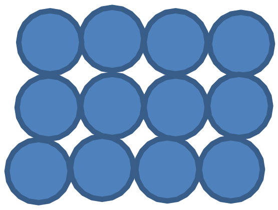
Pupils should recognise how to represent repeated addition problems using arrays.

Concrete



Pictorial





Abstract

3 groups of 4

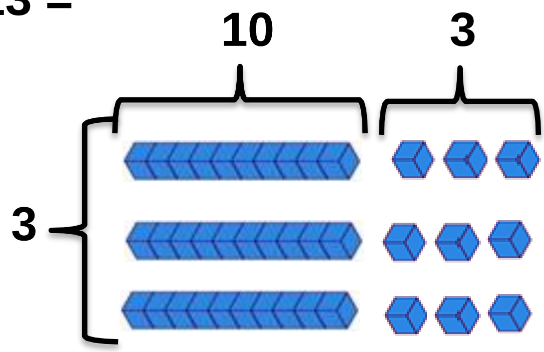
3 x 4

Stage 3

Pupils should establish their own understanding of how arrays can develop into the grid method for multiplication.

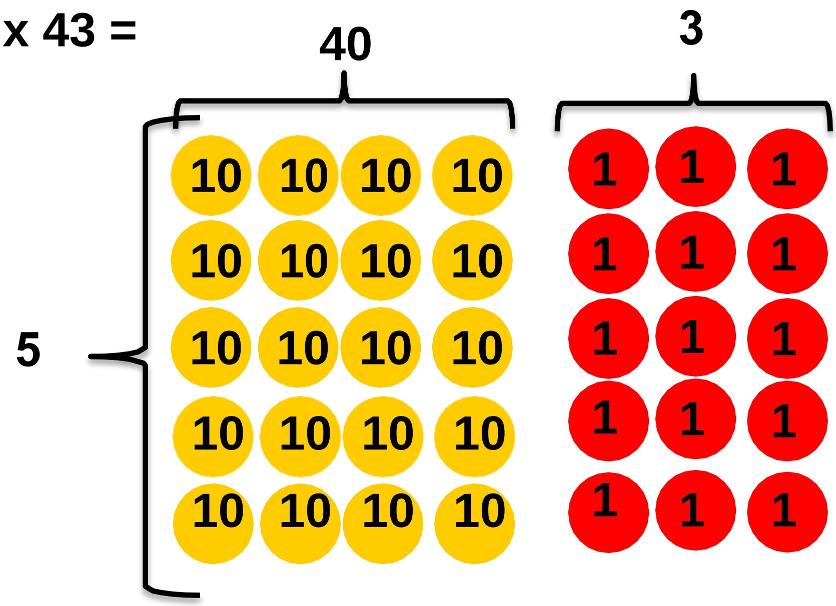
Concrete

3 x 13 =



Pictorial

5 x 43 =



Abstract

x	10	5
5	50	25

15 x 5 = 50 + 25 = 75

x	100	70	5
60	6000	4200	300
3	300	210	15

175 x 63 = 11025

Stage 4

Pupils should establish their own understanding of how the grid method can be converted into formal short and long multiplication written methods.

Short multiplication

2741 x 6

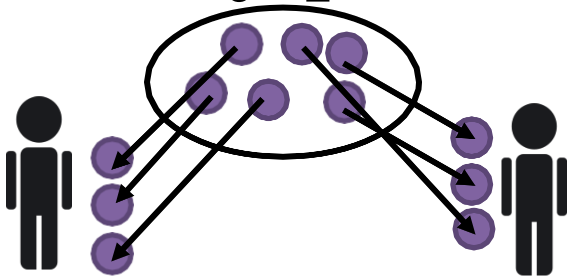
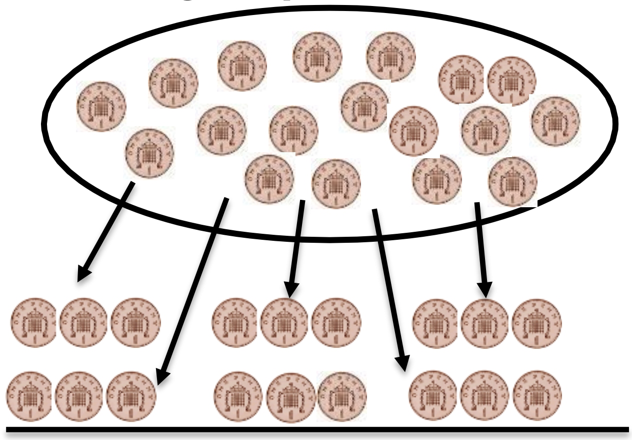
```
  2741
X    6
------
 16446
------
  4 2
```

Long multiplication

3124 x 26

```
  3124
X   26
------
 18744
 62480
------
 81224
------
  1 1
```

Structures of Division

Equal Sharing How many each Share equally between	**6 shared equally by 2** **6 ÷ 2** 
Inverse Multiplication How many lots of Share equally into groups	**18 ÷ 3** **18 divided into equal groups of 3s**  **6 groups of 3**
Ratio Comparison Inverse Scale factor	*Philip bought four times as much bread as Fred. If Philip bought 12 loaves how many does Fred have?* *Anna's trip to the seaside is three times as long as Ella's. If Anna took 3 hours how long does it take Ella?*

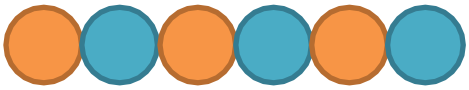
Stage 1

Grouping and Sharing

Concrete

Sharing

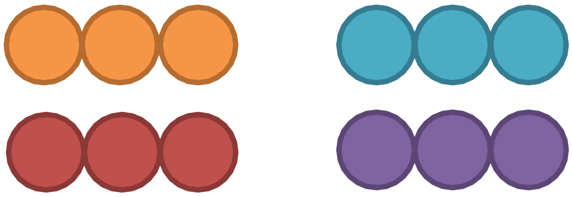
6 shared equally by 2



Each group as 3

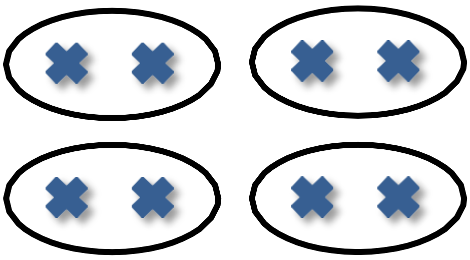
Grouping

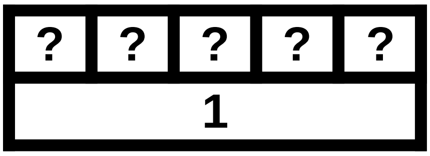
12 split into equal groups of 3



There are 4 groups

Pictorial





Abstract

6 shared between 3

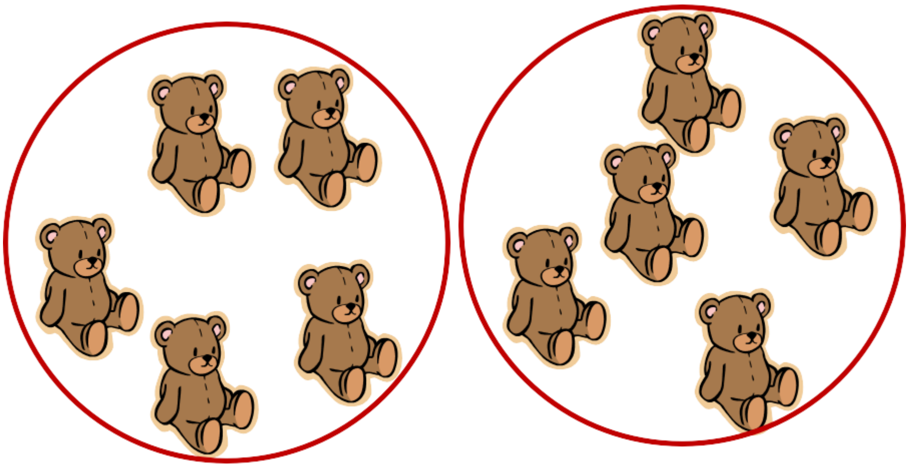
8 divided into 4 equal groups

Stage 2

Pupils should being to use repeated subtraction and addition with links to arrays and number lines.

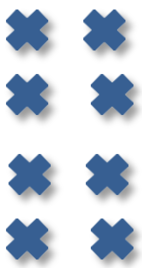
Concrete

10 divided by 5

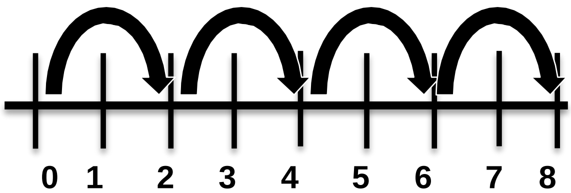


Pictorial

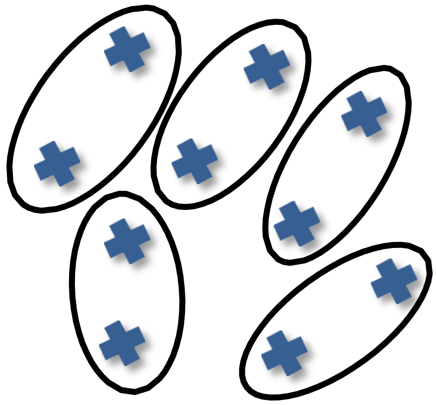
8 divided by 2



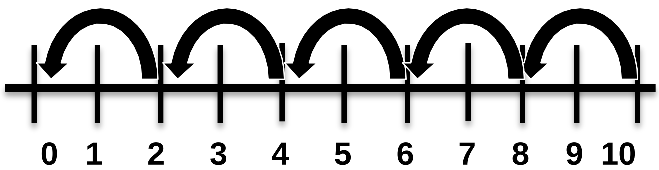
“Keep adding in 2s up to 8”



10 divided by 2



“Take away 2 each time”



Abstract

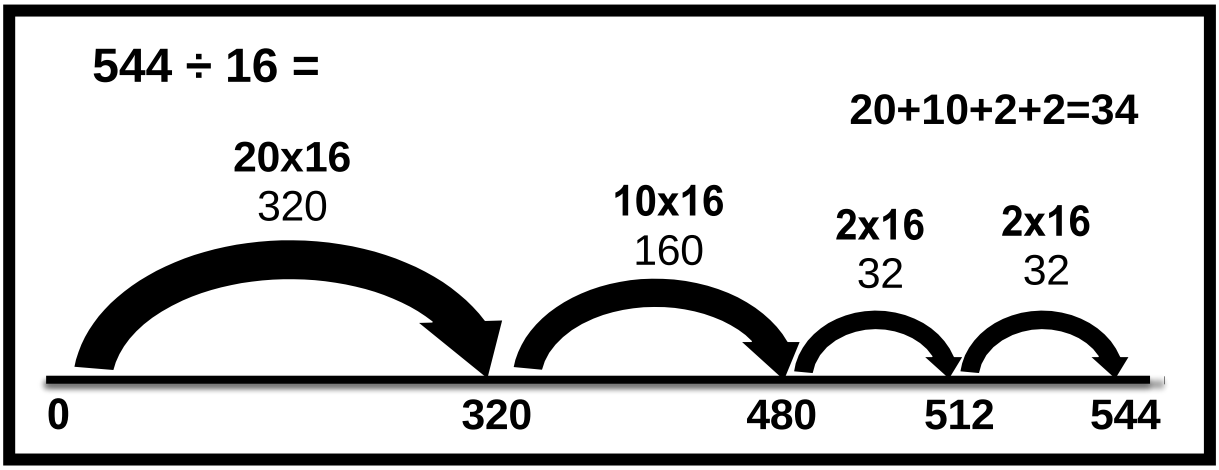
6 ÷ 3

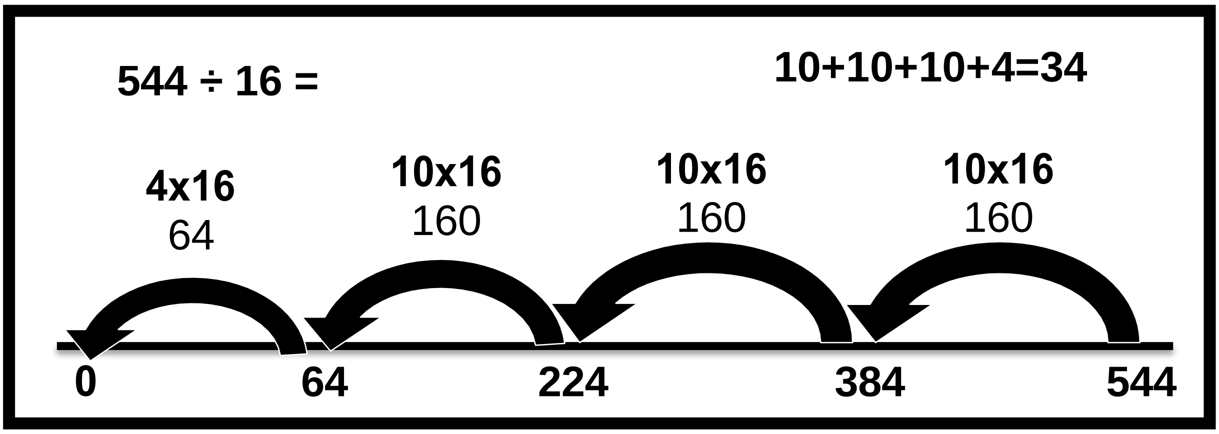
Pupils should be very secure in their understanding of the concept of division, including remainders, before moving onto the stage 3.

Stage 3

Chunking – using known number facts to support division.

Pictorial





Abstract

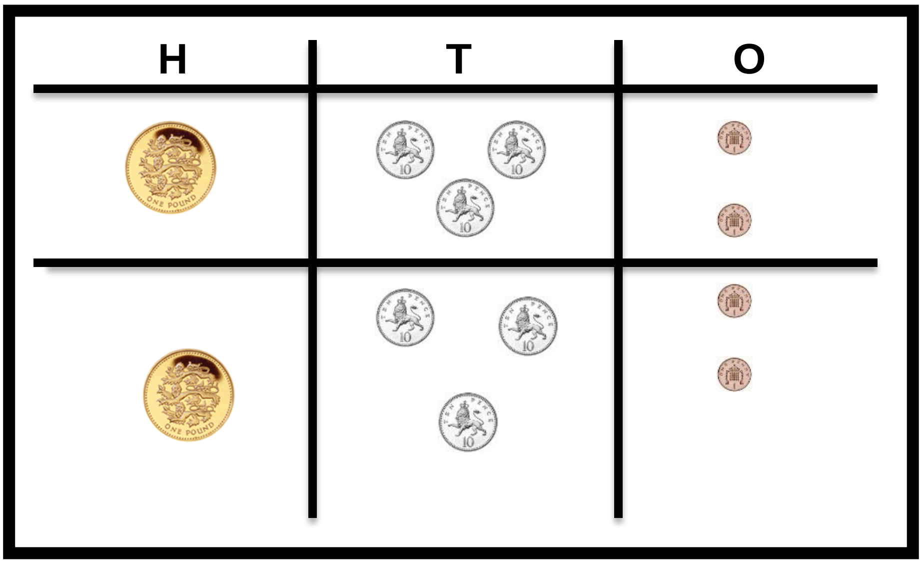
$$
\begin{array}{rl}
34 & \\
16\overline{)544} & \\
-\ 320 & (20 \times 16) \\
\hline
224 & \\
-\ 160 & (10 \times 16) \\
\hline
64 & \\
-\ 32 & (2 \times 16) \\
\hline
32 & \\
-\ 32 & (2 \times 16) \\
\hline
0 & \\
\hline
\end{array}
$$

Stage 4

Short and long methods for division, including remainders as whole number remainders, fractions, and by rounding, as appropriate for the context.

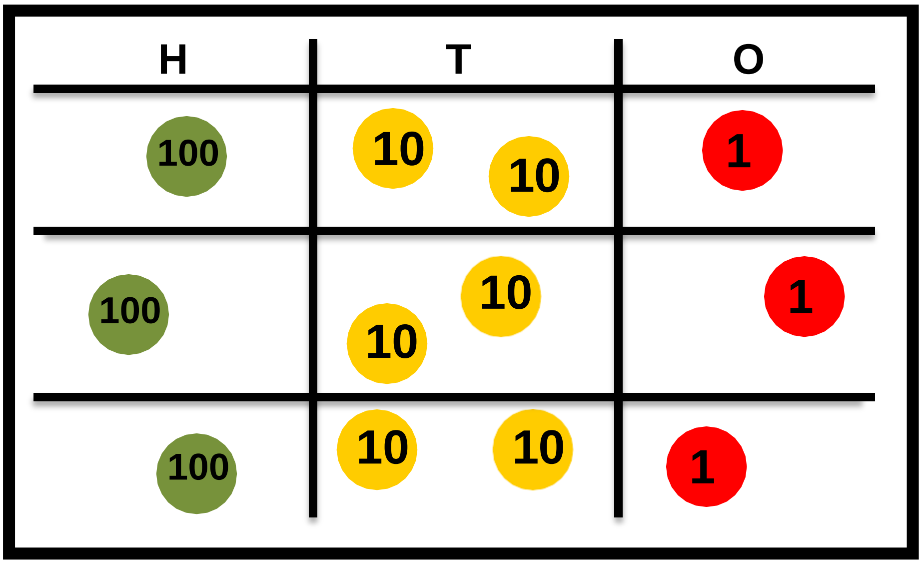
Concrete

264 ÷ 2 =



Pictorial

363 ÷ 3 =



Abstract

Short Division

$$\begin{array}{r|l} & 140 \\ \hline 4 & 5\,{}^{1}6\,0 \end{array}$$

Long Division

$$\begin{array}{r|r} & 23 \text{ r } 8 \\ \hline 24 & 560 \\ & -\,48 \\ \hline & 80 \\ & -\,72 \\ \hline & 8 \end{array}$$